The Boston Area Undergraduate Physics Competition

April 27, 2002

Name: ____________________________________________

School: __________________________________________

Year: _____________________________________________

Address: __________________________________________

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Do not turn this page until you are told to do so.

You have four (4) hours to complete this exam.

Please provide the information requested on this cover sheet. At the end of the exam, hand in this cover sheet with your solutions. You may keep the exam questions.

Show all relevant work in your exam books. Please write neatly. Partial credit will be given for significant progress made toward a correct solution.

You must be enrolled in a full-time undergraduate program to be eligible for prizes.
Each of the six questions is worth 10 points.

1. Two trapezoidal containers, connected by a tube as shown, hold water.

   (a) If the water in container $A$ is heated (causing it to expand), will water flow through the tube? If so, in which direction?

   (b) If the water in container $B$ is heated (causing it to expand), will water flow through the tube? If so, in which direction?

   (Assume that the containers do not expand when heated.)

Figure 1: Problem 1, Trapezoidal Containers
2. A mass, which is free to move on a horizontal frictionless plane, is attached to one end of a massless string which wraps partially around a frictionless vertical pole of radius $r$, as shown on Fig. 2 (top view). You hold on to the other end of the string. At $t = 0$, the mass has speed $v_0$ in the tangential direction along the dotted circle of radius $R$ shown.

Your task is to pull on the string so that the mass keeps moving along the dotted circle. You are required to do this in such a way that the string remains in contact with the pole at all times.

What is the speed of the mass as a function of time? Explain what happens in this system as time goes by. (Ignore any relativistic effects.)

Figure 2: Problem 2, Mass on a String

3. A brick is thrown (from ground level) at an angle $\theta$ with respect to the (horizontal) ground. Assume that the long face of the brick remains parallel to the ground at all times, and that there is no deformation in the ground or the brick when the brick hits the ground.

If the coefficient of friction between the brick and the ground is $\mu$, what should $\theta$ be so that the brick travels the maximum total horizontal distance before finally coming to rest?
4. A sheet of metal lies on a roof which is inclined at an angle \( \theta \). The coefficient of kinetic friction between the sheet and roof is \( \mu \) (where \( \mu > \tan \theta \)).

During the warmth of daytime, the sheet will expand slightly. And then during the nighttime it will contract. Let the coefficient of thermal expansion of the sheet be \( \alpha \), and let the difference in temperature between day and night be \( \Delta T \). Let the length of the sheet (from its upper edge to lower edge) be \( \ell \).

How far down the roof will the sheet move in one year if \( \theta = 30^\circ \), \( \mu = 1 \), \( \ell = 1 \) m, \( \Delta T = 10^\circ \text{C} \), and \( \alpha = 17 \cdot 10^{-6} \) \( \text{(C}^\circ)^{-1} \) (the \( \alpha \) for copper)? Assume uniform contact with the roof.

(Note: the change in length due to thermal expansion is \( \Delta L = \alpha L \Delta T \).)

5. A charged object generally induces an image charge when placed near a metallic plate. If the object moves, currents in the metal will lead to damping of its motion. Consider the following model for the dissipation: The image charge’s motion lags by time \( \tau \) behind the object’s motion.

(a) What applied force is necessary to sustain the motion of an object with charge \( q \) moving with constant velocity \( v \) parallel to an infinite metal plate, characterized by the time lag \( \tau \)? Let the distance from the plate be \( r \).

(b) Find the leading contribution (for small \( v \) and \( \tau \)) to the force calculated in part (a) in the direction of \( v \). What is the damping coefficient \( \gamma \) (which is defined by \( \mathbf{F} = -\gamma \mathbf{v} \))?

(c) What is the damping coefficient for motion perpendicular to the plate?

6. A small ball is attached to a massless string of length \( L \), the other end of which is attached to a very thin vertical pole. The ball is thrown so that it initially travels in a horizontal circle, with the string making an angle \( \theta_0 \) with the vertical.

As time goes on, the string will wrap itself around the pole. Assume that (1) the pole is thin enough so that the length of string in the air decreases very slowly, so that the ball’s motion may always be approximated as a horizontal circle, and (2) the pole has enough friction so that the string does not slide on the pole, once it touches it.

Find the difference in height between the point where the string is attached to the pole and the point where the ball eventually hits the pole.

Also, find the ratio of the ball’s final speed (right before it hits the pole) to its initial speed.