Boston Area Undergraduate
Physics Competition

April 22, 2000
Time: 4 hours

Each of the six questions is worth 10 points.

1. A thin stick with length L (and uniform mass density) is pivoted at a point P. The stick is held horizontal and then released. Where should P be located so that the stick swings down and passes through the vertical position in the minimum time?

2. (a) Two conducting infinite half-planes meet at a right angle. A charge q is brought in from rest at infinity, to a position (at rest) a distance d from each plane. What is the work done, W_{in}, to bring about this change?

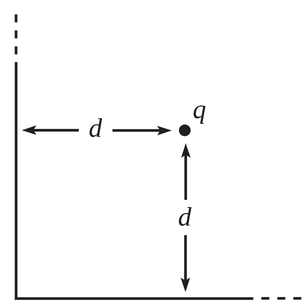

Figure 1: Problem 2

   (b) While the charge q is a distance d from each plane, the planes are changed from conducting to insulating (that is, the charges on them are no longer free to move). The charge q is then brought back out to infinity. What is the work done, W_{out}, to bring about this change?

   (c) What is the potential energy of the system of charges on the insulating planes?

3. Consider a planet which is made of matter with the same average density as the earth. Assume that the atmospheric pressure on the planet's surface is the same as on the earth's surface. Assume for simplicity that the planet's atmospheric temperature is independent of height, and that it is equal to the temperature at the surface of the earth. And assume that the composition of the atmosphere on the planet is the same as on the earth.

   What should the radius of the planet be, so that a light beam can travel in a circle around the planet, just above its surface?

   Note: You will need to use the fact that the index of refraction depends on the density of air according to $n(\rho) = 1 + \epsilon\rho$, where ϵ is a given constant.

   Give your answer in terms of:

R_E, the radius of the earth,

g_E, the acceleration at the surface of the earth,

P_E, the atmospheric pressure at the surface of the earth,

ρ_E, the atmospheric density at the surface of the earth, and

ϵ.

4. N identical balls lie equally spaced in a semicircle, on a frictionless horizontal table, as shown. The total mass of these balls is M. Another ball of mass m approaches the semicircle from the left, with the proper initial conditions so that it bounces (elastically) off all N balls and finally leaves the semicircle, heading directly to the left.

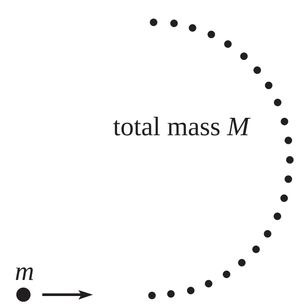

Figure 2: Problem 4

   (a) In the limit $N \to \infty$ (so the mass of each ball in the semicircle, M/N, goes to zero), find the minimum value of M/m which allows the incoming ball to come out heading directly to the left.

   (b) In the limiting case found in part (a), find the ratio of m's final speed to initial speed.

5. A turntable rotates with constant angular speed Ω. A ball rolls on it, without slipping. The ball has uniform mass density, so that its moment of inertia is $I = (2/5)MR^2$.

   Show that whatever the (non-slipping) initial conditions are, the ball will move in a circle (as viewed from the inertial lab frame). What is the frequency of this circular motion?

6. A rope of mass density σ hangs from a spring with spring-constant k. In the equilibrium position, the bottom part of the rope lies in a heap on the floor, and a length L is in the air. The top of the spring is held fixed.

   The rope is raised by a very small distance b and then released. What is the amplitude of oscillations, as a function of time?

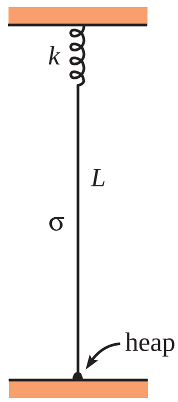

Figure 3: Problem 6

(Assume the following: (1) $L \gg b$, (2) the rope is very thin, so that the size of the heap on the floor is very small compared to b, (3) the length of the rope in the initial heap is larger than b, so that some of the rope always remains in contact with the floor, and (4) there is no friction of the rope with itself inside the heap.)