1997

The Boston Area Undergraduate

Physics Competition

April 26, 1997

Time: 4 hours

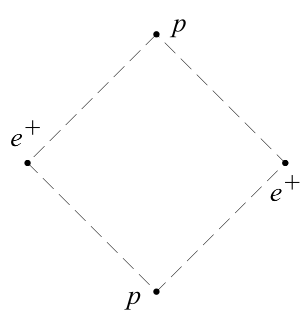
Each of the six questions is worth 10 points.

1. These two unrelated problems are each worth 5 points.

   (a) A block of ice containing a small air bubble floats in a pond. The ice melts. Does the water level go up, go down, or stay the same? Explain.

   Answer the same question for the situation where the air bubble is replaced by a small piece of lead.

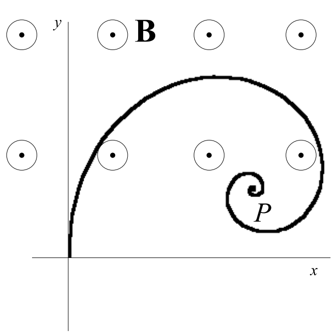
   (b) Two protons are held at opposite corners of a square, and two positrons are held at the other corners. The square has sides of length $\ell = 10^{-3}$ m.

   The particles are released. Give an approximate value (one significant figure is sufficient) for the speeds of the particles when they are very far apart.

   Notes: The mass of a positron is $m \approx 9.1 \cdot 10^{-31}$ kg, and the mass of a proton is $M \approx 1.7 \cdot 10^{-27}$ kg. Coulomb's Law says that the force between any two of these particles, separated by r meters, is equal to $2.3 \cdot 10^{-28}/r^2$ Newtons.

2. Let $\mathcal{R}_{y>0}$ be the region of space where $y > 0$. Let $\mathcal{R}_{y>0}$ contain a constant magnetic field $\vec{B} = B\hat{z}$ (see the figure below; $\vec{B}$ points out of the page).

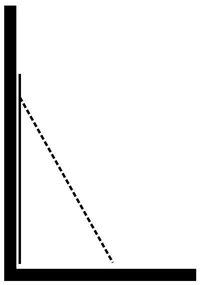
   

   A particle with charge q and mass m travels along the y-axis and enters $\mathcal{R}_{y>0}$ with speed v_0.

   Assume that in $\mathcal{R}_{y>0}$ the particle is subject to a friction force, $\vec{F}_f$, proportional to its velocity, i.e., $\vec{F}_f = -\alpha\vec{v}$. (You may ignore gravity.)

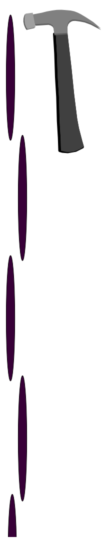
   Assume this friction force is large enough so that the particle will remain inside $\mathcal{R}_{y>0}$ at all times. The particle will then spiral in toward the point P. What are the coordinates of the point P?

3. A stick of length ℓ and uniform mass density per unit length leans against a frictionless wall. The ground is also frictionless.



The stick is initially held motionless, with its bottom end an infinitesimal distance from the wall. The stick is then released, whereupon the bottom end begins to slide away from the wall, and the top end begins to slide down the wall.
A long time after the stick is released, what is the horizontal component of the velocity of its center of mass?

4. This problem deals with rigid 'stick-like' objects of length $2r$, masses M_i, and moments of inertia $\rho M_i r^2$, where ρ is a numerical constant.

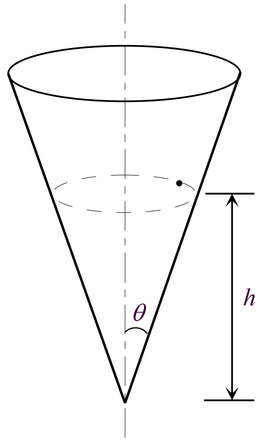


The center of mass of each stick is located at the center of the stick. (All the sticks have the same r, ρ, and relative mass distribution. Only the masses differ.) Assume $M_1 \gg M_2 \gg M_3 \gg \cdots$.
The sticks are placed on a horizontal frictionless surface. The ends overlap a negligible distance, and the ends are a negligible distance apart.
The first (heaviest) stick is given an instantaneous blow (as shown) which causes it to translate and rotate. (The blow comes from the side of stick #1 on which stick #2 lies [the right side, as shown in the figure].) Depending on the value of ρ, the first stick may strike the second stick, which will then strike the third stick, and so on. Assume all collisions among the sticks are elastic.
Depending on the value of ρ, the speed of the nth stick will either (1) approach zero, (2) approach infinity, or (3) be independent of n, as $n \to \infty$.

What is the special value of ρ corresponding to the third of these three scenarios? Give an example of a stick having this value of ρ.

Note: You may work in the approximation where M_1 is infinitely heavier than M_2, which is infinitely heavier than M_3, etc.

5. (a) (2 points) A fixed cone stands on its tip, with its axis in the vertical direction.

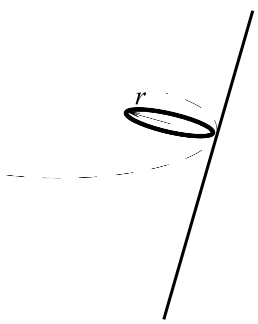


When viewed from the side, the cone subtends an angle of 2θ. A particle of negligible size slides on the inside surface of the cone. This surface is frictionless.

Assume conditions have been set up so that the particle moves in a circle at a height h above the tip.

What is the frequency, ω, of this circular motion?

(b) (8 points) Assume now that the surface has friction, and a small ring of radius r rolls on the surface without slipping.

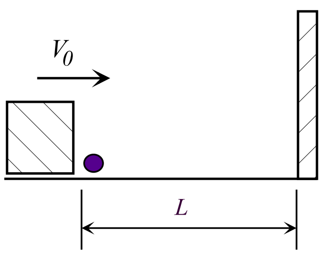


Assume conditions have been set up so that (1) the point of contact between the ring and the cone moves in a circle at a height h above the tip, and (2) the plane of the ring is at all times perpendicular to the line joining the point of contact and the tip of the cone.

What is the frequency, ω, of this circular motion? How does it compare to the answer in part (a)?

Note: You may work in the approximation where r is much smaller than the radius of the circular motion, $h \tan\theta$.

6. A block with very large mass M slides on a frictionless surface towards a fixed wall.



The block's speed is V_0. The block strikes a particle with very small mass m (and negligible size), which is initially at rest at a distance L from the wall. The particle bounces elastically off the block and slides to the wall, where it bounces elastically and then slides back toward the block. The particle continues to bounce elastically back and forth between the block and the wall.

(a) (7 points) How close does the block come to the wall?

(b) (3 points) How many times does the particle bounce off the block, by the time the block makes its closest approach to the wall?

Note: In both parts (a) and (b), you may assume $M \gg m$, and you need only obtain *approximate answers, valid to leading order in* m/M. (In other words, pick quantities of, say, $M = 10$ kg, $m = 1$ g, $L = 1$ m, and $V_0 = 1$ m/s, and obtain approximate answers to the above two questions.)