The Boston Area Undergraduate Physics Competition

April 18, 1998

Name: ______________________________

School: ______________________________

Year: ______________________________

Address: ______________________________

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e-mail: ______________________________

Phone: ______________________________

Do not turn this page until you are told to do so.

You have four (4) hours to complete this exam.

Please provide the information requested on this cover sheet. At the end of the exam, hand in this cover sheet with your solutions. You may keep the exam questions.

Show all relevant work in your exam books. Please write neatly. Partial credit will be given for significant progress made toward a correct solution.

You must be enrolled in a full-time undergraduate program to be eligible for prizes.

1998
Boston Area Undergraduate
Physics Competition

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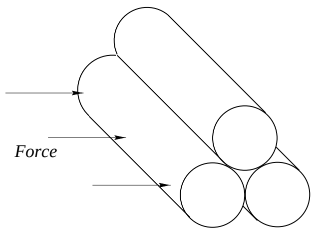
Time: 4 hours

Each of the six questions is worth 10 points.

1. Three identical cylinders are arranged in a triangle as shown, with the bottom two lying on the ground.

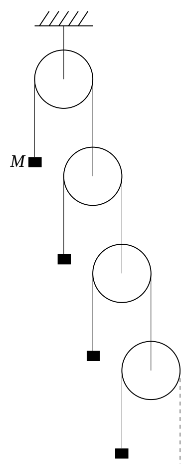


The ground and the cylinders are frictionless. You apply a force (directed to the right) on the left cylinder.
What are the minimum and maximum accelerations you may give to the system in order for all three cylinders to remain in contact with each other?

2. A charged particle sits at the center of a circular region containing a magnetic field, $\vec{B}$. The field $\vec{B} = \vec{B}(r)$ depends only on the radial position r, and it is perpendicular to the plane of the circle. The total magnetic flux through the circle is zero.

   The particle is given a kick. Show that if the particle leaves the circular region, then at the instant it leaves, its velocity points in the radial direction.

3. Consider the infinite Atwood's machine shown in the figure.



A string passes over each pulley, with one end attached to a mass and the other end attached to another pulley. All the masses are equal to M, and all the pulleys and strings are massless.

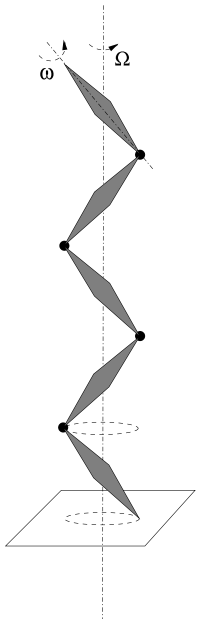
The masses are held fixed and then simultaneously released. What is the acceleration of the top mass?

(You may define this infinite system as follows. Consider it to be made of N pulleys, with a non-zero mass replacing what would have been the $(N+1)$st pulley. Then take the limit as $N \to \infty$. It is not necessary, however, to use this exact definition.)

4. Each edge of an icosahedron is a 1Ω resistor. Find the effective resistance between two adjacent vertices.

   (An icosahedron consists of 20 equilateral triangles. It has 12 vertices and 30 edges, with 5 edges meeting at each vertex.)

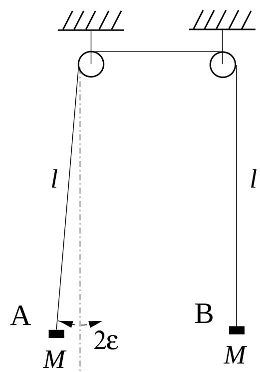
5. Consider the setup of N identical cylindrically symmetric tops in the figure.



The bottom one rests on a frictionless table. Each top is connected to the one above it by a free pivot. The inclination angles of the tops are the same. The center of mass of each top is at the midpoint of its symmetry axis.

You wish to set up a very slow circular precession of the tops, where the CM of each top stays fixed while the ends travel in circles. The angular speed of the top top is ω. Find the angular speeds of all the other tops as functions of ω. (You may work in the approximation where these speeds are very large.)

6. Two masses, A and B, each have mass M and are attached to the ends of a massless string.



The string passes over a set of massless pulleys of negligible size. The masses are at rest at a distance ℓ from the pulleys.

Mass A is then given a very small horizontal kick, so that it initially swings back and forth with amplitude ϵ (where $\epsilon \ll \ell$).

It turns out that after a very long time, one of the masses will eventually rise up and hit its pulley.

(a) (3 points) Which mass hits its pulley?

(b) (7 points) What is the speed of mass B, right before the hitting of the pulley occurs?

(Throughout this problem, work in the approximation where $\epsilon \ll \ell$.)